RESEARCH OF THE CHARGE LOAD INFLUENCE OVER THE TUMBLING MILL CHARACTERISTICS THROUGH GENERAL UTILITY FUNCTION

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ABSTRACT

The experimental assessment of ball mills type MCB 4.5x6 (МШЦ 4,5х6) operating in copper ore processing plant and SAG mill 8.5x5.3 operating in gold ore processing plant are presented. The experimental results are statistically processed and some statistical regression models are estimated. It was developed a general utility function in terms of generalized grinding usefulness functions based on proper regression models. In parallel, some analytical calculations about ball fill coefficient gave the ability to connect measurement and calculated values, so they are used in connection with experimental data. A mathematical function about general utility function in dependence of ball fill coefficient, productivity, mill rotational speed and chamber diameter with special note about charge coefficient is also developed. Results are summarized in some useful practical implementation routines.

Keywords: ball mill, SAG mill, experimental analysis, purpose function, general utility function.

INTRODUCTION

Tumbling mills are the main type mills used in grinding process for mineral processing in wide range of industrial technologies - in silicate industry there are used chamotte mills, in carbonates industry there are used clinker-cement mills, in mineral processing - ball mills, autogenous and semi-autogenous mills. Ball mills in mineral processing are widely used machines with combination with rough and middle stage pre-crushing. The other main group - SAG mills are usually in chain only with rough crushing. Typical product from grinding circuit with ball mills and SAG mills is in the range of 75 - 95 % of 50 - 80 micrometers. The typical tumbling mills in silicate, carbonate and building materials industries operate in dry chamber conditions while the mineral processing machine are mainly with the wet chamber - media conditions. The detailed comparative studies [1, 6, 9 - 12] of different grinding types are rare and are interesting field of discussion.

In the current study are examined ball mills used in some of the mineral processing facilities in Bulgaria. More specifically MCB 4.5x6 ball mill (МШЦ 4.5х6) working with copper-sulphide ore. SAG 8.5x5.3 mill that is processing gold ore with significant ball load are also examined. Both types of examined tumbling mills are operating in closed circuits with water transport and hydro-cyclone classification systems.

where: Q, t/h is an average tonnage input-output productivity per hour (productivity debit); k, % is the quality per examined grain size class, for example k_75 is quality in percent for class -75 µm;

P - mill net power draw; q = Q · k, t / h - specific productivity of examined grain size class; q_{th} - theoretical specific productivity;

\[ e = \frac{P}{q} = \frac{P}{Q \cdot k}, kW \cdot h / t \]

- specific energy consumption in production of examined grain size class;

Table 1. Theoretical specific productivity and energy consumption of the examined machines.

<table>
<thead>
<tr>
<th></th>
<th>Q</th>
<th>k</th>
<th>P</th>
<th>e_{th}</th>
</tr>
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<tbody>
<tr>
<td>t/h</td>
<td>%</td>
<td>kW</td>
<td>kW·h/t</td>
<td></td>
</tr>
<tr>
<td>MCB 4.5x6</td>
<td>200</td>
<td>85</td>
<td>2500</td>
<td>14.71</td>
</tr>
<tr>
<td>SAG 8.5x5.3</td>
<td>400</td>
<td>85</td>
<td>6000</td>
<td>17.65</td>
</tr>
</tbody>
</table>
Theoretical specific energy consumption: 

- theoretical specific energy consumption; 
- rpm is rotational mill speed; 
- mill critical speed according to chamber inner diameter; 
- chamber diameter, as it is shown at Fig. 1; 
- diameter between lifter peaks; 
- free high over horizontal milling medium to the top of the chamber; 
- free high to the center of the chamber; 
- milling medium filled area in chamber; 
- torque arm of moving chamber media (charge); 
- charge displacement angle in case of rotating drum; 
- specific fill coefficient; 
- specific quality characteristic coefficient; 
- m$^3$/h - back water debit is the transportation water debit; 
- the steel ball mass added in mill operation; 
- kN - the chamber media weigth force; 
- specific (angular) speed coefficient, calculated with ratio between mill operating speed and mill critical speed $\psi = \frac{n}{n_{cr}}$.

This research is revealing the connection between the mill specific productivity and chamber parameters, technological and power parameters with instruments of analytical research, statistical analysis and mathematical modeling. The idea is following the simple chain connection diagram: chamber diameter → chamber fill → operating power → mill productivity.

**Theory**

Chamber diameter and the diameter between lifters are in complex connection with ball and slurry charge realizing change of arm and charge displacement angle. Connection between those parameters is quite complicated [7, 3, 11 - 13], so theoretical equation was settled: eq. (1) or eq. (2) in general form of Arbiter and Harris equation [13]. This equation does not take in consideration some of the valuable factors but it is in easy applicable form for further analysis.

Charging calculation scheme (Fig. 1(a)) and dimension accession are done by multipoint sizing with manual measuring gear and where it was possible laser scanning system is used (Fig. 2), which has 3 mm accuracy in the studied chamber volume of the mill [2, 15]. The mill charge (medium) level and the specific fill coefficient control are done when the mill is steady. The data for chamber free height and chamber diameter, according to Fig. 1, are measured and thus specific fill coefficient is calculated.

The mill power draw, the specific quality characteristic, the debit productivity and the ancillary factors: back water debit, diameter between lifters and addition steel ball mass are also counted during the measuring period through technological control system of examined machine. The model data form CAD, as it is shown at Fig. 3, is used in conjunction with the mill measurements (Fig. 2) increasing the comparison accuracy in calculated chamber fill area, volume and corresponding specific mill charge.

Theoretical base of ball mill power is evaluated by the following formula [7, 12 - 13]:

$$P = G_w \cdot g \cdot a \cdot \sin(\alpha) \cdot D \cdot \frac{\pi \cdot n}{60}, \quad kW$$

or

$$P = 0.5134 \left( \rho_w \cdot \frac{\phi}{100} \cdot \frac{\pi}{4} \cdot D^2 \cdot L \right) \cdot n \cdot k_{sp} \cdot D \cdot a, \quad kW$$

where $\phi = \frac{V_{GM}}{V}$, % is specific charge, 

- D, m - chamber effective diameter;

Fig. 1. Chamber charge scheme with main geometrical parameters used in steady state - a); rotating state - b); empty chamber geometry c).
- \( n \), \( \text{rpm} \) - mill rotational speed;
- \( k_{TF} = \sin(\alpha) \) – torque factor coefficient figures charge media (slurry and ball) bed displacement factor;
- \( \rho_w, \ t / m^3 \) – specific average bulk (slurry and ball) weight;
- \( a = 9 \cdot 10^{-3} \cdot (96.7 - \varphi) \), \( m \) – charge (slurry and ball) center distance;

constant simplification value: \( 0.5134 = \frac{\pi \cdot 60}{60} \)

The charge density depends on the ball fill coefficient, the slurry density and the lump percentage or feed size [2, 4, 6, 7, 9, 11 - 13] thus being a complex factor. If empirical data is used for specific bulk weight \( \rho_w = 4.6 \ t / m^3 \) and \( k_{TF} = \sin(\alpha) = 0.67 \), which are used as empirical values, but they have to be discussed separately, then the formula is simplified to:
The calculation of this dependence shows the characteristics of the function, which is quadratic to the specific charge coefficient and cubic to chamber effective diameter. The technological review of the dependencies that characterize the process in the mill is conducted by different method. The mill is considered as a black box input-output object as it is shown in Fig. 5 [5, 8, 9, 14]. In the current case the experimental data is obtained in industrial working equipment, so the experiment is a passive multifactor one and the values are measured by industrial measuring system with control measuring gages.

RESULTS AND DISCUSSION

The used examination methodology of connection between mechanical and technological parameters is figured in the following steps. Mill mechanical regime parameters are defined by three main factors:
- power consumption;
- specific charge coefficient;
- specific (angular) speed - ratio $\psi = n/n_{CS}$.

Since in the process are involved specific speed factors, which cannot be precisely determined, accordingly MCB ball mill is with non-adjustable driver speed, it was chosen approach to register correlations between specific fill coefficient and power draw. Based on the gathered data quadratic dependencies have been revealed in following form:

$$\varphi = A_1 \cdot P + A_2 \cdot P^2$$  \hspace{1cm} (7)

as all of the revealed dependencies fulfill the statistical criteria for significance and adequacy. The observed data for specific fill coefficient to mill power draw dependency are given in Table 2, as the shown models have multiple correlation coefficient $R^2$ is over 95%. The graphical representation of those models are shown in Fig. 6 with points of intersection between mill power draw and specific fill coefficient from achieved statistical models and observed data.

Table 2. Specific fill coefficient to power dependency from MSB ball mill data.

<table>
<thead>
<tr>
<th>$\varphi$ range</th>
<th>$\varphi(P_{\text{min}})$</th>
<th>$\varphi(P_{\text{mid}})$</th>
<th>$\varphi(P_{\text{max}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0233799</td>
<td>0.0000023</td>
<td>34.53</td>
</tr>
<tr>
<td>2</td>
<td>0.0186998</td>
<td>0.0000009</td>
<td>33.75</td>
</tr>
<tr>
<td>3</td>
<td>-0.0006767</td>
<td>0.0000083</td>
<td>34.01</td>
</tr>
<tr>
<td>4</td>
<td>0.0282288</td>
<td>-0.0000051</td>
<td>34.41</td>
</tr>
<tr>
<td>5</td>
<td>0.0095378</td>
<td>0.0000034</td>
<td>33.40</td>
</tr>
<tr>
<td>6</td>
<td>0.0281969</td>
<td>-0.0000053</td>
<td>34.01</td>
</tr>
<tr>
<td>7</td>
<td>0.0452425</td>
<td>-0.00000137</td>
<td>36.48</td>
</tr>
<tr>
<td>8</td>
<td>0.0183695</td>
<td>-0.0000007</td>
<td>35.36</td>
</tr>
</tbody>
</table>

\begin{equation*}
P = 11.07 \cdot 10^{-3} \cdot (96.7 - \varphi) \cdot \varphi \cdot D^3, \hspace{0.5cm} kW \hspace{1cm} (6)
\end{equation*}

Fig. 5. Mill technological block diagram.

Fig. 6. Intersection between mill power draw and specific fill coefficient.
Next step in this study is planning of the experiment with selection of purpose functions and control factors. These specific purpose functions have been chosen: productivity, energy consumption, and specific steel balls consumption. It must be noted that this current study does not have any initial information on the proper mathematical representation of the model sought [5, 8, 14].

With such experiment setting it was conducted testing of the following purpose functions:

- the specific productivity, prodused quantity per unit of time:
  \[ Y_1 = q_{pu} = \frac{Q}{100}, \text{t} / \text{h} \]  
  (8)

- the specific energy consumption:
  \[ Y_2 = e = \frac{P}{q_{pu}}, \text{KW} \cdot \text{h} / \text{t} \]  
  (9)

- the specific steel ball consumption per unit productivity
  \[ Y_3 = q_{sb} = \frac{M}{q_{pu}}, \text{kg} / \text{t} \]  
  (10)

The process control factors are limited due to the fact passive multifactor experiment is been conducted.

- \( X_1 = Q_{55}, \text{t} / \text{h} \) - feed (class -5 mm) from 1-st stage milling;
- \( X_2 = Q_{10}, \text{t} / \text{h} \) - feed (class -10 mm) from crushing plant;
- \( X_3 = P, \text{KW} \) - mill power draw;
- \( X_4 = M_b, \text{t} / \text{h} \) - mass of additional steel balls;

After the data is registered, sorted and counted the developed mathematical models are statistically processed, as they are examined by the following criteria: multiple correlation coefficient \( R^2 \) over 95 %, maximal value of Fisher criterion F-ratio (three-digit number and higher), probability index (p-value) for each of the coefficients less than 5 %.

After the result of this model processing and a few iterations of the model selection, models of type placed bellow were selected:

for specific productivity:

\[ Y_1 = A_3 \cdot X_2 \cdot X_3 + A_4 \cdot X_2^2, \text{t} / \text{h} \]  
(11)

For each machine mathematical model it is calculated with its numerical characteristics which fulfill the criteria above. Comparative graphics between the observed values and the mathematical model are shown in Fig. 7.

The specific energy consumption is:

\[ Y_2 = A_5 \cdot X_2 + A_6 \cdot X_3 + A_7 \cdot \sqrt{X_3}, \text{KW} \cdot \text{h} / \text{t} \]  
(12)

This type of models fulfills the selection criteria again and in Fig. 8 is shown comparative graphics for both of the machines.

Examination following such chain have a disadvantage in the preference of the final result to one of the functions. One way of avoiding such a disadvantage is to formulate the generic utility function. Generic utility function requires norming of the factors and purpose functions done by the following formula:

\[ \eta_j \left( x^{(u)} \right) = \frac{k_j \left[ X_j \left( x^{(u)} \right) - X_j^{\text{fit}} \right]}{X_j^{\text{max}} - X_j^{\text{min}}} \]  
(13)

where: - \( X_j^{\text{fit}} \) (futile) the most useless value of
normed parameter;
- coefficient “k” is: $k_j = +1$, when the increasing of $X_j$ is useful, and $k_j = -1$ when decreasing of $X_j$ is useful;
- $\eta_j$ - is the new normed factor or function;
- $X_j$ - the old (non-normed) factor or function;
- $X_j^{\text{min}}$ – the minimal value of normed parameter;
- $X_j^{\text{max}}$ – the maximal value of normed parameter;
- $x_u$ – point of measurement or serial number of evaluated data;
- $j$ - index of factors or functions.

After the norming the averaging general utility function without weight coefficients is defined as:

$$\Phi_{av} (x) = \frac{1}{m} \sum_{j=1}^{m} \eta_j (x)$$  \hspace{1cm} (14)

In the current paper general utility function by specific productivity, specific energy consumption and specific steel balls consumption is presented as follows:

$$\Phi_1 (x) = \frac{1}{3} (\eta_{q_{sw}} + \eta_e + \eta_{q_{sl}})$$  \hspace{1cm} (15)

After a few iterations of the general utility function models are done, the functions are developed as the requirements for significance and adequacy are fulfilled. The models are investigated in initial form as follows:

$$\Phi_1 = A_{11} \cdot X_2 + A_{12} \cdot X_3 + A_{13} \cdot X_2 \cdot X_3$$  \hspace{1cm} (16)

The search formula shown in eq. 16 is selected to have similarity to the mathematical representation with the model for net power draw shown in eq. 6. All the models presented here are selected as the best models through criteria maximal multiple correlation coefficient $R^2$. According to this criteria some of them are quite complicated in mathematical formulation and physical meanings. The comparative graphics shown at Fig. 7

Fig. 8. Comparative graphics (y=x) between the mathematical model (x axis) and the observed values(y axis) for $Y_2$ function: a) MCB, b) SAG.

Fig. 9. Comparative graphics (y=x) between the mathematical model (x axis) and the observed values(y axis) for general utility $\Phi_1$ function: a) MCB, b) SAG.
to Fig. 9 are also presenting the best models behavior.

CONCLUSIONS

By some practical reasons in the elucidation of the mathematical-physical formula meaning, the second model for utility function $\Phi_2$ is defined to follow the general form of theoretical formula (eq. 6) and it is represented by surface diagram on Fig. 10 with some fixed factors. Selected general utility function depends on four factors and it is not applicable for diagram representation. General utility function $\Phi_2$ is as follows:

$$\Phi_2 = A_{21} \cdot X_4 \cdot X_1^3 + A_{22} \cdot X_2^2 + A_{23} \cdot X_2 \cdot X_4 \quad (17)$$

or

$$\Phi_2 = A_{21} \cdot Q \cdot D^3 + A_{22} \cdot \varphi^2 + A_{23} \cdot n \cdot Q \quad (18)$$

General utility function $\Phi_2$ allows major conclusions about its dependence from the four factors. Particularly, one of them is specific fill which is representative about ball load in milling chamber. General utility function $\Phi_2$ is statistically significant, adequate and relevant only in the examined interval of control factors. The general utility function $\Phi_2$ shows that the ball load is increasing function value with its own growth so it can be used as a positive control factor in process control. In both examples the particular weight of specific fill or ball load is different, but the influence is in the same direction over the function value.
REFERENCES


15. T. Hrisova, Methodology for determining the replacement period for lifter bars, SIMPRO, 8, 6, 2018.