DISTRIBUTION OF LINEAR PRESSURE OF THIN-SHEET ROLLING ACROSS STRIP WIDTH

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ABSTRACT

In the article the influence of metal transverse displacements in the zone of plastic deformation at thin-sheet rolling on alignment of lengthening coefficients across width of the rolled strips is considered. Mathematical calculations of the function of the rolling pressure distribution per unit length in the strip width on the basis of the Jourdain variational principle are represented. An analysis is made of the behavior of the rolling pressure distribution per unit length in the strip width depending on the relative reduction, friction coefficient, amplitude of unevenness of entrance velocities of metal in the zone of deformation, width of the rolled strips, work rolls radius, and also frequency of entrance velocities unevenness changing.

Keywords: thin-sheet rolling, transverse displacements, pressure distribution, Jorden variation principle.

INTRODUCTION

It was found by the researchers of the thin-sheet rolling, that unevenness of elongations (and, accordingly, output stresses) across strip width, measured after rolling, is less than ones calculated in supposition of plane state deformation.

It is connected with the self-alignment mechanism due to transverse displacements of metal in the zone of plastic deformation [1 - 3].

The spreading when rolling strips and sheets promotes alignment of elongations across width, but also when rolling without spreading at uneven reduction on width, transverse displacements of metal take place [4 - 9].

The scheme of influence of transverse displacements of metal in the deformation zone on reduction of unevenness of elongations across strip width at thin-sheet rolling without spreading is represented in Fig. 1.

Initial billet with $2B$ width, the cross-sectional profile of which consists of three areas $A$, $B$ and $C$, having thickness $h_{0A}$, $h_{0B}$ and $h_{0C}$, enters in the plastic deformation zone with the speed $v_0$, and at the exit of the deformation zone the rolled strip has the same thickness $h_1$.

If rolling was carried out according to the scheme of flat deformation, the elongation and output speed of metal in areas $A$, $B$ and $C$ is defined by the following ratios:

$$
\lambda_A = \lambda_C = \frac{h_{0A}}{h_1} = \frac{h_{0C}}{h_1}; \\
v_{x1} = v_{x3} = v_0 \frac{h_{0A}}{h_1} = v_0 - \frac{h_{0C}}{h_1}; \\
\lambda_B = \frac{h_{0B}}{h_1}; \\
v_{x2} = v_0 \frac{h_{0B}}{h_1}. \\
$$

The borders of areas $A$, $B$ and $C$ are defined by the planes parallel to edges of a strip and passing through the line segments $aa$ and $bb$ (Fig. 1a).
The distribution of metal speeds across strip width in case of the plane deformation state is shown in Fig. 2 by the polygonal dash-dotted line with an amplitude $\Delta v_x = \Delta$. The transverse displacements of metal with a speed $v_y$ in the deformation zone changes borders of areas on value $S$, and areas A, B and C pass to areas A', B' and C', thus separating planes pass through pieces of straight lines $a'a'$ and $b'b'$ (Fig. 1b).

In the presence of transverse displacements of the metal in the zone of deformation the longitudinal speeds of the metal in areas A $v'_{x1}$ and C $v'_{x3}$ increase, but area B $v'_{x2}$ decreases relatively to the speeds in the case of plane deformation state:

$$v'_{x1} > v_{x1}; \quad v'_{x3} > v_{x3}; \quad v'_{x2} < v_{x2}$$

(2)

The distribution of longitudinal metal speeds across the strip width in the presence of the transverse displacements of the metal in the deformation zone is shown in Fig. 2 by the solid line with amplitude $\Delta v'_x = A'$; the lengthening coefficients of the metal in the respective areas, calculated as the ratio of the input and output thicknesses remain unchanged.

The influence of transverse displacements of the metal in the deformation zone to reducing of non-uniformity of the elongations across the strip width is taken into account by the coefficient $\rho$:

$$\frac{\Delta \lambda(y)}{\lambda} = \rho \left[ \frac{\delta h_0(y)}{h_0} - \frac{\delta h_1(y)}{h_1} \right]$$

(3)

where $\Delta \lambda(y)$ and $\lambda$ are the value of the current non-uniformity of the elongations and value of average elongation across the strip width, $\delta h_0(y)$ and $h_0$ - the value of the current transverse thickness variation and average thickness of rolled stock, $\delta h_1(y)$ and $h_1$ - the value of the current transverse thickness variation and average thickness of the strip, $0 < \rho < 1$ - coefficient taking into account the influence of transverse displacements of the metal in the zone of plastic deformation; in plane state of deformation $\rho = 1$.

The longitudinal stress unevenness causes loss of the flat shape strip [10 - 15] and at the exit of the deformation zone is expressed in accordance with the formula:

$$\Delta \sigma_{yav}(y) = -E \frac{\Delta \lambda(y)}{\lambda}$$

(4)

where $E$ is the module of elasticity of the material of the strip.

To determine the dependence of the coefficient $\rho$
on the main rolling parameters, the energy balance of the process of rolling on a smooth barrel is considered.

On an entrance to the zone of plastic deformation, unevenness of height deformation, longitudinal tensions and speeds of metal flow across strip width, is noted [16 - 20].

Since the value of deformation and speed unevenness is significantly small in comparison with their average estimates, generally this unevenness can be expressed as unevenness of metal speeds on an entrance to the zone of plastic deformation.

**Theoretical studies**

The input unevenness of speeds was described as \( f'(y) < 1 \), and output \( -\phi'(y) < 1 \) (Fig. 3). It was assumed that the spreading when rolling is absent, i.e. \( f(0) = f(B) = 0 \) and \( \phi(0) = \phi(B) = 0 \), \( B \) – half-width of a strip.

For deformation zone we will assume a model of the rigid-plastic medium with elastic external zones [21-23], i.e. we consider that a metal, not possessing an elasticity in the deformation zone, at once acquires it on an exit from the deformation zone.

Applying the Jorden variation principle to defined the deformation zone:

\[
\delta\left(\int \int \int T dv - \int \int \int P ds + \sum_{i=1}^{n} \int \int \int \tau_i |\Delta v_i| ds\right) = 0 \tag{5}
\]

where \( T \) and \( H \) are the shear stress and deformation rates intensities; \( P \) and \( v \) – external forces operating on the borders of the deformation zone, and corresponding to their speeds of displacements; \( \tau_i \) – shear yield point of strip material; \( \Delta v_i \) – a jump of speeds on \( i \)-th shear surfaces; \( \delta \) – variation symbol.

The first integral represents the power of internal resistance, the second – the power of external forces acting on the deformation zone borders: sliding friction forces between rolls and a strip, a forward and back tension, the third - the power of cut forces.

One of the components of a forward tension power is called as the power spent for accumulation of potential energy by a strip.

The equations for speeds of displacements and deformations of metal are:

\[
v_x(y) = v_x \left[ 1 + f'(y) \frac{h_x - h_1}{\Delta h} + \phi'(y) \frac{h_0 - h_x}{\Delta h} \right],
\]

\[
v_y = -\frac{v_x h'}{\Delta h}, \tag{6}
\]

\[
x_x = \frac{\partial v_x}{\partial x} = \xi_x \left[ 1 + \frac{\phi' h_0 - f' h_1}{\Delta h} \right],
\]

\[
x_y = v_x \frac{h'}{h_x}, \quad \xi_y = -\xi_x - \xi_z.
\]
where $\Delta h = h_0 - h_1$ is the reduction in thickness, $v_x$ – average value of speed of metal on width in cross-section $x$.

The power of internal resistance is:

$$\frac{N_{\text{int}}}{\tau_s} = \int_0^h B l \, H \, dy \, dx \, dz,$$

where

$$H = \sqrt{\xi_x^2 - \xi_y^2} \xi_z$$

(7)

The power of sliding friction forces between rolls and a strip:

$$\frac{N_{sl}}{\tau_s} = 4\mu \int_0^B dy \int_0^l \sqrt{(v_x - v_{roll})^2 + v_y^2}$$

(8)

where $\mu$ is constant of friction between work rolls and a strip.

The power spent for accumulation of potential energy by a strip is:

$$\frac{N_{\text{pot}}}{\tau_s} = v_1 h_1 \int_0^B \frac{\sigma_y^2}{2E} \, dy$$

(9)

where $\sigma_y = \sigma_0 + \sigma_{\text{out}}$; $\sigma_0$ – average value of specific forward tension on width; $\sigma_{\text{out}} = \sigma_{\text{rest}} = -\phi \sigma$ (we assume that the output stresses become the value of the residual unchanged).

To find the function of distribution of line rolling pressure across the strip width, the method of determining the mechanical coupling reaction is applied, based on mechanical coupling rejection, i.e. the function is transferred from the category of predetermined into the category of variable.

Following this method, the work rolls have given small vertical translational velocity $W^*$ (Fig. 3).

At the same time in the second integral equation (1) (the power of the external forces acting on the borders of the deformation zone), the power of the line rolling pressure $p(y)$ will appear:

$$\frac{N_{p}}{\tau_s} = \int_0^B W^* \, p(y) \, dy$$

(10)

We neglect the power of cut forces, since at the sheet rolling, they are small and do not affect the overall balance of power.

By summing the powers calculated with the help of the expressions (7-10), we obtain an expression for the functional $F(\phi', \phi, W^*, W)$.

Solving the system of equations Euler-Lagrange and directing $\eta_0$ to zero, i.e., restoring a dropped connection, we obtain an analytic expression for the derivative of the distribution function of pressure per unit length of rolling:

$$\frac{\partial F}{\partial \phi'} = \frac{d}{dy} \left[ \frac{\partial F}{\partial \phi'} \right] = 0$$

(11)

$$\frac{\partial F}{\partial W} - \frac{d}{dy} \left[ \frac{\partial F}{\partial W} \right] = 0$$

$$p(y) = 4\mu v_1 h_1 (\phi - f) \ln \left[ \frac{2 \sqrt{(1 - m^2) + a^2(\phi - f)^2} - 2 m^2 
+ a^2(\phi - f)}{a^2(\phi - f)} \right] + f'' \ell \left( \frac{h_1}{\Delta h} + \frac{2}{3} \right) - \phi'' \ell \left( \frac{h_0}{\Delta h} + \frac{2}{3} \right),$$

(12)

where $a = \frac{2v_1 h_1}{\ell \Delta h}$; $m_1 = \frac{x_i}{\ell}$; $X_i$ – coordinate of neutral section.

RESULTS AND DISCUSSION

Using expression (7) the function of distribution of linear pressure of rolling across width was investigated at the following conditions:

- the radius of the work rolls $R = 300, 450, 600 \text{ mm}$;
- input thickness $h_0 = 4, 0 \text{ mm}$;
- shear yield point of strip material $\tau_s = 50 \text{ N/mm}^2$;
- module of elasticity of material of strip $E = 10^5 \text{ MPa}$;
- relative reduction $\varepsilon = 40; 60; 80 \%$
- constant of friction $\mu = 0.1, 0.2, 0.3$;
- half-width of a strip $B = 500; 700; 900 \text{ mm}$;
- input unevenness of speeds $f''(y) = A \cos \left( k \pi \frac{y}{B} \right)$

$k = 1, 2, 3$;

unevenness amplitude $A = -0.0005 \div +0.0005$;
- elastic contact flattening and bending of work rolls are absent.

Fig. 4 shows the dependence of the distribution of linear pressure of rolling, which is divided by the yield...
point, across width, on the value of relative reduction.

The calculations were performed under the following conditions: $R = 450$ mm; $\mu = 0.2$; $B = 700$ mm; $k = 1$; $A = 0.0005$.

It is shown that with increasing of relative reduction, the rolling force increases as it should be, and the unevenness of the distribution of linear pressure of rolling, decreases.

This is due to the fact that the conditions for transverse displacements of the metal in the zone of plastic deformation become more favorable with increasing of reduction.

The results of calculation of influence of coefficient of friction on distribution of linear pressure for the terms of $R = 450$ mm; $\varepsilon = 60\%$; $B = 700$ mm; $k = 1$; $A = 0.0005$ are presented on Fig. 5.

With the increase of coefficient of friction, the conditions for transversal displacements of metal get worse, and, as a result, the degree of unevenness of distribution of linear pressure increases.

Fig. 6 shows the dependence of the distribution of linear pressure on the value of unevenness of metal speeds in the input of the zone of plastic deformation.

The calculations were performed under the following conditions: $R = 450$ mm; $\varepsilon = 60\%$; $\mu = 0.2$; $k = 1$; $B = 700$ mm.

In the areas of the strip, where, in the direction of rolling, the metal flows with speeds exceeding average on a width, longitudinal tensions of compression operate; accordingly, in these areas linear pressure is higher than in the strip areas, where the metal flows with speeds lower than the average on width.

All curves intersect in a point corresponding to the zero of the function

$$f'(y) = A \cos\left(\frac{\pi y}{B}\right)$$

The distribution of linear pressure at $A = 0$, is uniform, that fully corresponds to physical sense.

Fig. 7 shows the dependence of the strip width on distribution of linear pressure on the strip width.

The wider the strip, the less favorable conditions for the transverse displacements of metal in its middle part are, accordingly, the more uneven distribution of linear pressure, and also, the narrower the strip, the better the conditions for the transverse displacements of metal, and the less uneven is the distribution of linear pressure.

The calculations were performed under the following conditions: $R = 450$ mm; $\varepsilon = 60\%$; $\mu = 0.2$; $k = 1$; $A = +0.0005$.

All curves also intersect in a point corresponding to the zero of the function

$$f'(y) = A \cos\left(\frac{\pi y}{B}\right)$$

Fig. 8 shows the dependence of the strip width on distribution of linear pressure on the radius of work roll.

The calculations were performed under the following conditions: $B = 700$ mm; $\varepsilon = 60\%$; $\mu = 0.2$; $k = 1$; $A = +0.0005$.

With the increase of the work roll radius amplitude of unevenness distribution slightly reduced (in accord-
Fig. 6. Influence of the unevenness amplitude on linear pressure.
1 - $B = 900$ mm; 2 - $B = 700$ mm; 3 - $B = 500$ mm.

Fig. 7. Dependence of the strip width on the distribution of linear pressure.
1 - $R = 500$ mm; 2 - $R = 700$ mm; 3 - $R = 900$ mm.

Fig. 8. Influence of the work rolls radius on linear pressure.
1 - $k = 1$; 2 - $k = 2$; 3 - $k = 3$.

Fig. 9. Influence of periodicity of the unevenness of the metal input flow velocities on the distribution of linear pressure.
$B = 700$ mm; $\varepsilon = 60$ %; $\mu = 0.2$; $A = +0.0005$.

With the increase of frequency of unevenness change of metal flow speeds in the zone of deformation, amplitude of unevenness of distribution of linear pressure diminishes.

ance with the length of the deformation zone).

Fig. 9 shows the dependence of the distribution of linear pressure on periodicity of the unevenness function of the input metal flow speeds. The calculations were performed under the following conditions: $R = 450$ mm;
CONCLUSIONS

As a result of theoretical calculations based on the idea of transverse displacements of the metal in the zone of deformation, derivative of the distribution of linear rolling pressure across the strip width are explicitly obtained. The behavior of the distribution of linear rolling pressure across the strip width, depending on various factors is studied.

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REFERENCES

15. V.N. Shinkin, A.P. Kolikov, Engineering calculations for processes involved in the production of large-diameter pipes by the SMS Meer technology, Metallurgist, 55, 11-12, 2012, 833-840.
20. S.M. Belskiy, Some effects of the application of an