SUPERHEAT REMOVAL IN CONTINUOUS CASTING MOLD

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ABSTRACT

Methods and mathematical equations of superheat removal, shell growth, temperature distribution in continuous casting mold are considered. They can be used for continuous casters engineering and control systems.

Keywords: continuous casting, continuous casting mold, solidification equation, mathematical model, superheat.

INTRODUCTION

The distinctive feature of continuous casting process is a high intensive heat transfer in the “slab-mold” system and relatively short time of slab residing in the mold. At the same time only initial slab shell solidifies in the continuous casting (CC) mold. As a rule, the volume of this shell is much less than the volume of liquid metal participating in casting process. Therefore the shell forming in the mold is the starting phase of the whole CC slab solidification.

It is clear that to study the kinetics of CC slab solidification is very important to take into account few factors that can be missed as inessential in ordinary casting processes analyzing. First of all it concerns the superheat influence on the shell growth. In analysis the sand or metal mold gravity casting when the ingot billet completely solidifies in the mold the superheat usually is taking into account according with wide known “enthalpy Chvorinov’s rule”. This considerably simplifies analysis and mathematical modeling of the solidification. But in the continuous casting mold the shell growth starts immediately at the meniscus (vertical casting) [1] or in the so called “transient zone” (horizontal casting) [2] and such assumption of superheat “smearing” along solidification front can cause errors. Therefore it is not quite acceptable. Mathematical model of the process has to consider direct superheat influence on the shell solidification in the CC mold.

MATHEMATICAL MODEL

The proposed mathematical model of superheat influence on the shell solidification in the CC mold is based on the wide applying of Successive Approximations Method.

To get mathematical equations one introduces simplifications which are usual in the CC slab solidification theory [3]. Also the parabolic function of heat transfer coefficient from the solidification surface to the mold cooler (water) is accepted [4]:

\[ k = (k_0 - k_e) \left( 1 - \frac{\tau}{T} \right)^m \]  

\( k \) - heat transfer coefficient; \( k_0 \) - heat transfer coefficient at the meniscus; \( k_e \) - heat transfer coefficient at the mold outlet; \( \tau \) - time below meniscus; \( T \) - time of slab residing in the mold; \( m \) - parabolic exponent of heat transfer coefficient distribution curve.

In addition is accepted that general superheat residue, \( Q_{sh} \), remaining in the slab core at the time moment, \( \tau \), is proportional to the general heat flux in the mold. The last state was proved and expressed mathematically by I. Pugachev in [5]:

\[ Q_{sh} = Q_o \left( 1 - \frac{k^* \tau}{k^{*} \cdot T} \right) \]  

\( Q_o \) - general superheat flux, delivered into the mold for one time unit; \( F \) - cross section slab surface; \( w \) - casting speed; \( \rho' \) - liquid metal density; \( c' \) - liquid metal specific heat; \( \theta_o \) - casting pouring temperature; \( \theta_s \) - solidification temperature; \( k^* \) - average integral heat transfer coefficient; \( k^{*} \) - average integral heat transfer coefficient at superheat.
time removal; \( Th \) - complete superheat removal time.

Average integral heat transfer coefficients \( k^- \) and \( k^-_{Th} \) are calculated by integrating the equation (1) in corresponding time intervals \( 0 \leq \tau \leq 0+Th \):

\[
k^- = \frac{(k_0 + m \cdot k)}{(m+1)}
\]

(3)

To use (2) to set up and solve energy conservation equation of solidification slab in CC mold one should know values, \( k^- \) and, \( Th \). These values one can get from the simple heat conservation balance of elementary CC slab section [5].

\[
k^- \cdot \vartheta_s \cdot dS \cdot d\tau = - c^- \cdot \rho^- \cdot (X - \xi^-) \cdot dS \cdot d\vartheta
\]

(4)

dS - finite difference CC slab cooling surface; \( X \) - slab thermal geometrical module (according with Chvoronoj’s rule); \( \xi^- \) - average integral slab shell thickness solidified in regarded time interval; \( d\vartheta' \) - elementary liquid slab core temperature. If divide variables and integrate equation (4) in time interval, \( 0 \leq Th \), and in temperature interval from, \( \vartheta_0 - \vartheta_s \), one gets:

\[
Th = \frac{c^- \cdot \rho^- \cdot (X - \xi^-)}{k^-_{Th} \cdot \vartheta_s} (\vartheta_0 - \vartheta_s)
\]

(5)

It is impossible to use equation (5) for direct, \( Th \), calculation because of indefinite value of \( \xi^- \). That is why it is necessary to introduce further simplifications.

As a first approximation it is accepted that, \( \xi^- = 0 \). Such assumption for \( Th \) calculation has a real ground. As mentioned above the shell volume in the mold is much less than the volume of slab core fluid. This is especially typical for starting length of CC mold where the main superheat quantity is extracted. That is why the influence of \( \xi^- \) on the \( Th \) value is insignificant. Nevertheless the introduced assumption is accepted as the first and highly rough approximation which needs further specification. Taking into account the above assumption the equation (5) can be written as:

\[
Th = \frac{c^- \cdot \rho^- \cdot X}{k^-_{Th} \cdot \vartheta_s} (\vartheta_0 - \vartheta_s)
\]

(6)

The \( Th \) calculation for equation (6) is performed by Successive Approximations Method (every next iteration value awards by corresponding index - 1, 2, 3…). As the first approximation one can accept \( k^-_{Th} = k^-_f \), where \( k^-_f \) - average integral heat transfer coefficient in time interval \( 0 \leq T \). \( k^-_f \) is calculated by substitution into equation (3) value, \( k = ke \). Accepted value, \( k^-_{Th} \) is put in (6) and calculates the superheat time removal as second approximation \( Th^2 \). The same order is used in making third approximation and so on until one gets necessary precision of \( k^-_{Th} \) and \( Th \). The calculations reveal that 3 - 4 iterations are quite sufficient to get result when for example subsequent \( k^-_{Th} \) value differs from previous value less than 1 W m\(^{-2}\)K\(^{-1}\). It is very high precision if taking into account that heat transfer coefficient in the CC mold is the value of \( 3^{\text{rd}} - 4^{\text{th}} \) order.

The heat conservation balance with taking in mind the above assumptions is:

\[
dQ = dQ_{sh} + dQ_r + dQ_c
\]

(7)

dQ - finite differential heat flux from the slab to water cooling mold at the time \( d\tau \); \( dQ_{sh} \) - finite differential superheat; \( dQ_r \) - finite differential latent heat; \( dQ_c \) - finite differential specific heat of the slab shell. The solving of equation (7) can be expressed as the law of shell growth in CC mold:

\[
x = \frac{n(n+1)\lambda r}{2ck\delta s} \left[ \sqrt{1 + \frac{n(n+1)\lambda c\rho'}{cck\delta s}} \left[ k^-\vartheta_s \cdot \tau - F w p'c (\vartheta_0 - \vartheta_s)(1 - \frac{k^- \cdot \tau}{k^-_{Th} \cdot Th}) \right] - \frac{n(n+1)\lambda r}{2ck\delta s} \right] \cdot \xi
\]

(8)

\( \xi \) - shell thickness; \( c \) - specific heat of solid metal; \( \rho \) - density of solid metal; \( \lambda \) - thermal conductivity of solid metal; \( r \) - latent heat; \( n \) - parabolic exponent of temperature distribution curve in the slab shell.

It is necessary to explain that calculation of function \( \xi(\tau) \), according with equation (8) is executed in time interval \( 0 \leq Th \), only and under the condition of \( Th \leq T \). The consequent calculation of function \( \xi(\tau) \) in the time interval \( Th - T \) (under the same condition) is executed according with equation for slab shell solidification in the CC mold without superheating, has been published in the article [4].
\[ \xi = \left[ \frac{n(n+1)\lambda r}{2ck\theta s} \right]^2 + \frac{n(n+1)\lambda}{c_0 k\theta s} k^- \cdot \tau - \frac{n(n+1)\lambda r}{2ck\theta s} \]  
(9)

Calculation method of function \( \xi(\tau) \) under condition \( \text{Th} \leq \text{T} \) needs separate consideration because of its relative complexity and that is why will be discussed later.

And now when function \( \xi(\tau) \) is known, it is possible to make second approximation for more precise calculation of value \( \text{Th} \) but with adequate equation (5). For this purpose it is necessary to calculate the average value of shell thickness \( \xi^- \) in the mold with the help of function \( \xi(\tau) \) from (8). There are two ways for that. The first way is to solve equation (8) numerically with any interpolation method. The second way is by integrating the function (8) directly in time interval 0 - \( \text{Th} \). It is not necessary to explain the first way. So, let consider the second one in details.

It is impossible to integrate function \( \xi(\tau) \) in form of equation (8). That is why one offers to interpolate this function by another approximated function with standard tabular integral or which is easily transformed to the standard tabular integral. Best of all the parabolic type function meets these conditions:

\[ \xi = a \cdot \tau^b \]  
(10)

\( a \) is proportion ratio coefficient; \( b \) is parabolic exponent. Magnitudes of \( a \) and \( b \) are easy determined by corresponding numerical substitutions into (8) and further logarithmization. The choice of parabola is natural. It is easy to notice that on term \( b = 0.5 \) the equation (9) is transformed into wide known so called “square root law” which has played a remarkable role in solidification theory developing [6] including continuous casting slab solidification [7]. In this case the coefficient \( a \) acquires “solidification constant” sense. It is easy to find

\[ \xi^- = \frac{a \cdot \text{Th}^b}{b + 1} \]  
(11)

And now by substitution known \( \xi^- \) to equation (5) it is possible to calculate values of \( \text{Th} \) and \( k^-_{\text{Th}} \) from equations (3) and (5) as a second approximation. After that according with new \( \text{Th} \) and \( k^-_{\text{Th}} \), it is possible to calculate \( \xi(\tau) \) and value \( \xi^- \) as the second approximation. Using these data one can make calculations in third approximation and so on. The sequence of operations is the same as in the first approximation. The experience of calculations shows that two interpolations are quite enough to get very high precision. At the first sight the offering method of superheat flux influence on continuous casting slab solidification looks “bulky” but the modern computer engineering makes its application sufficiently effective.

The experimental researches and analysis of received mathematical relationships show that there are three possible cases and therefore three options of using offered method for shell calculation in the CC mold.

I. \( \text{Th} < \text{T} \)

Superheat is completely removed before slab exit from the CC mold. This case more often takes place in continuous casting practice as it provides the most stability of the process. The calculation algorithm in this case is the following. The time interval 0 - \( \text{T} \) is divided into two subintervals: 0 - \( \text{Th} \) and \( \text{Th} - \text{T} \). Within the first limit the shell thickness calculation is executed according with equation (8); within the second - according with (9).

II. \( \text{Th} = \text{T} \)

This second case is a particular case of the first one. The shell thickness calculation is executed completely according with the equation (8). But the probability that the time interval of superheat removal will be exactly equal to the time of the slab stay in CC mold is miserable.

III. \( \text{Th} > \text{T} \)

Superheat is not completely removed in the CC mold. This third case is the most difficult from the mathematical point of view. But Successive Approximations Method permits to solve this problem too. The calculation algorithm in this case is following.

As the first approximation function \( \xi(\tau) \) is calculated by equation (9) without taking into account the superheating. This new function is approximated by parabolic type curve (10) and on its basis the \( \xi^- \) is calculated in (11) but within the time interval 0 - \( \text{T} \). Further this value of \( \xi^- \) is put up into (5) to get \( \text{Th} \). But it is accepted that average value of heat transfer coefficient \( k^-_{\text{Th}} \) within the time interval 0 - \( \text{Th} \) is treated as equal to \( k^- \). Moreover the magnitude of \( \text{Th} \), calculated according with (5), is a relative value in this case because superheat removal of the slab beyond the CC mold is determined by cooling rate on the air in spray zone region. Then the magnitudes of \( k^-_{\text{Th}} \) and \( \text{Th} \) should be put into equation (2) and composed the heat conservation balance of sys-
It is necessary to explain that the integrating of equation (7) in this case is made in time interval 0 - T but not in 0 - Th as in the first two cases. After that this new function, $\xi(x)$ is accepted as the second approximation i.e. it is interpolated as parabolic curve (10) and coefficient $a$ and parabolic exponent $b$ are determined. On that base average shell thickness $\bar{x}$ is calculated and then all cycle of operations is repeated. The calculation cycles are made until one gets necessary precision.

Case III is not widely used in continuous casting practice because the cooling rate beyond the CC mold falls down sharply and the shell’s breakout is possible. However this case needs to be analyzed because it can be useful in spray zone parameters defining: if from total superheat one subtracts the superheat extracted in the CC mold within the time interval 0 - T, one can get the superheat quantity remaining in the slab beyond the CC mold. And just that quantity can be used as initial data to determine the second cooling parameters.

The further analysis of the proposed mathematical model results in to the very important conclusion. As it follows from equations (8) and (9) with relatively slow casting speed and “small superheat” (relatively low pouring temperature) i.e. on condition $Th < T$ the slab shell thickness in the CC mold exit does not depend on pouring temperature. I.V. Samarasekera and J.K. Brimacombe came to the same conclusion in their previous work [8] by researching experimentally the CC shell thickness in the mold exit depending on the pouring temperature. It is possible to explain this as it seems paradoxical fact by the following. The influence of superheating affects most intensive on the shell growth in the beginning phase of the process only. Herein after it fades out quickly. As the result the shell thickness in the CC mold inlet region will be less than the slab shell thickness solidifying without superheating or with smaller superheating but with the same other process parameters. Therefore in this region the thermal resistance of the heat transfer from the solidification front to the mold cooler (water) will be less because of the less shell thickness and the less shrinkage interface gap size. That’s why if in the starting stage the solidification rate of overheated liquid metal in the CC mold in every fixed time moment is lower than solidification rate of the CC slab without or smaller superheat but to the time moment $Th$ it becomes higher (because of smaller thermal resistance). This provides to equalizing of the CC mold outlet shell thickness of continuous casting process without superheating and with “reasonable” overheating.

If one subjects to the similar analysis the case $Th > T$ (this takes place in higher superheating degrees and higher casting speeds) as it follows from the proposed mathematical model: the more superheating - the less outlet shell thickness.

CONCLUSIONS

The offered model of superheat flux influence on the continuous casting slab solidification in the CC mold includes all cases of the continuous casting process and therefore can be used in determining the optimal CC technological parameters and also in developing the continuous casting engineering and control systems.

REFERENCES