

EFFECTS OF TEMPERATURE AND CONCENTRATION DEPENDENT VISCOSITY ON ONSET OF CONVECTION IN POROUS MEDIA

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ABSTRACT

The instability theory is applied to diffusion-convection phenomena in porous media, where the viscosity of the oil varies due to gas dissolution. An important application of this theory refers to the case where the diffusion-convection is employed as an EOR (Enhanced Oil Recovery) technique in oil reservoirs.

This paper presents results on the onset of Rayleigh-Bénard and Darcy-Bénard-Marangoni convective motions of a Boussinesq fluid taking into account its temperature dependent viscosity and surface tension. Numerical simulations are carried out using the Volume of Fluid (VOF) model, while that of Darcy is applied to the mathematical formulation in evaluating the flow structure and the heat transfer in a two-dimensional fluid porous layer. The results obtained show the trend observed by Hashim and Wilson.

Keywords: Rayleigh-Bénard (RB) convection, Darcy-Bénard-Marangoni (DBM) convection, onset of convection.

INTRODUCTION

Darcy-Bénard (DB) convection is a type of natural convection determined by the unstable vertical density difference in a horizontal fluid saturated porous layer heated from below. The DB convection is extensively studied [1 - 3] because of its importance referring to many scientific, engineering and technological applications such as oil and gas recovery and underground contaminant transport.

The buoyancy-induced flow in a cavity when the heat transfer comes from below leads to patterns of convection cells. However, apart from the buoyancy forces, the convective instability can also result from the surface tension at the free surface contact with the gas known as Darcy-Marangoni (DM) convection. The Marangoni convection in a liquid saturated porous media is discussed [4, 5] in details. Rudraiah and Prasad study [6] the effect of Brinkman boundary layer on the onset of convection in a porous layer driven by surface tension gradients. In addition, Nield suggest a composite fluid and porous layer model for the study of Marangoni convection in a porous layer [7]. Desai, Lebon and Hennenberg investigate [8] the coupled capillary and

gravity driven instability in a fluid layer overlying a porous one. Using the Brinkman's model to describe the flow in the porous medium they determine the critical Rayleigh and Marangoni numbers for the convection onset. Shivakumara et al. [9] obtain an exact solution for the onset of the surface tension driven convection in superposed fluid and porous layers using the Darcy momentum law.

The aim of this paper is to simulate a porous layer saturated with liquid which is subjected to Darcy-Bénard, Darcy-Marangoni and Darcy-Bénard-Marangoni (DBM) convection.

GOVERNING EQUATIONS

A fluid confined in a porous medium is considered with the assumption that the following linear dependencies hold for the density and the viscosity:

$$\rho = \rho_0 [1 - \alpha(C_0 - C)] \quad (1)$$

$$\frac{\mu}{\mu_0} = 1 - \beta(C - C_0) \quad (2)$$

It is assumed that Boussinesq approximation is valid and the fluid density varies linearly with the temperature in the buoyancy force term:

$$\rho = \rho_0[1 - \alpha_T(T - T_0)] \quad (3)$$

where α_T is the thermal expansion coefficient and ρ_0 is the density at $T=T_0$. The fluid viscosity is assumed to vary linearly with the temperature in correspondence with:

$$\frac{\mu}{\mu_0} = 1 - \beta_T(T - T_0) \quad (4)$$

Moreover, the surface tension is also a function of temperature

$$\frac{\sigma}{\sigma_0} = 1 - \tau(T - T_0) \quad (5)$$

The governing equations for the variable viscosity conditions are formulated [10] as follows:

The Continuity equation

$$\nabla \cdot \mathbf{v} = 0 \quad (6)$$

The equations of motion

$$\frac{\mu}{K} \mathbf{v} = -\nabla P + \rho \mathbf{g} + \mu \nabla^2 \mathbf{v} + \nabla \mu \cdot \nabla \mathbf{v} + \nabla \mathbf{v} \cdot \nabla \mu \quad (7)$$

The equation of diffusion

$$\mathbf{v} \cdot \nabla C = D_e \nabla^2 C \quad (8)$$

Wooding defined [11] the Rayleigh number for a viscous liquid of variable density in a porous medium. The relation he advances is:

$$R = \frac{d \rho}{dz} \frac{g K d^2}{\mu D_e} \quad (9)$$

where K is the permeability of the porous medium, d is tube diameter, while D_e is the effective diffusivity of the dissolved material.

NUMERICAL DETAILS

The governing equations were discretized by ap-

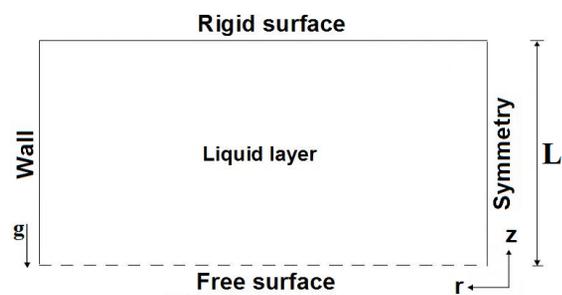


Fig. 1. Schematic presentation of the porous medium and the corresponding coordinate system.

plying the second-order upwind scheme for the volume fraction and the power law scheme for the momentum and energy equations. The VOF approach was employed to simulate the heat transfer and the fluid flow in the porous media.

As shown in Fig. 1, the simulation area refers to a liquid saturated porous layer of thickness L and two lateral walls, an impermeable on the left and a symmetrical on the right. The bottom boundary is rigid, while the top surface, which is in contact with the gas, is treated as a free surface. A temperature difference of ΔT is maintained between the top and bottom boundaries of the porous layer.

RESULTS AND DISCUSSION

The flow structure and the heat transfer due to Darcy-Bénard, Darcy-Maragioni and Darcy-Bénard-Maragioni convection are discussed.

Streamlines

Fig. 2 shows the streamlines that are formed due to DB and DBM convection. As it can be seen, the convection loops in case of DBM convection are bigger than those in presence of DB convection, illustrating thus the surface tension effect. In addition, the greater size of DBM convection loops corresponds to a smaller number of loops.

Heat transfer

Fig. 3 shows the average Nusselt number calculated for a cavity subjected to heat transfer from below. As it is seen, the increase of the thermal Rayleigh number results in increase of the average Nusselt number. In addition, it shows that the highest average Nusselt number is obtained in case of DBM convection. Besides, the Nusselt number obtained in presence of DB convection is higher than that obtained for DM convection.

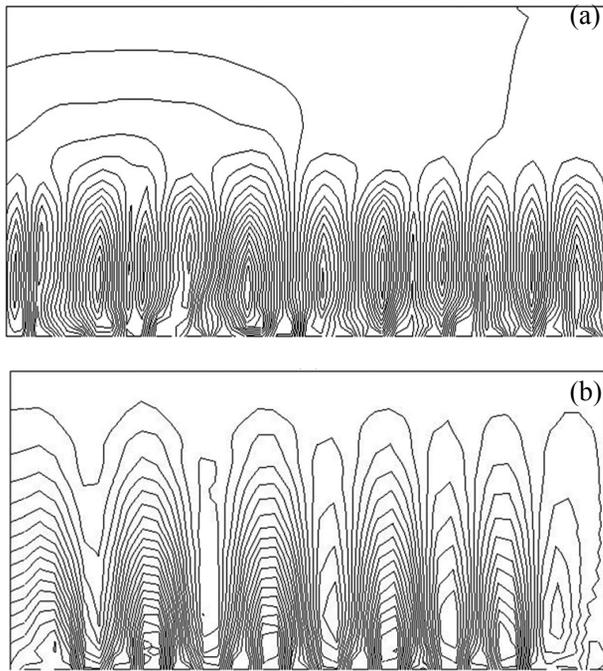


Fig. 2. Streamlines due to DB (a) and DBM (b) convection.

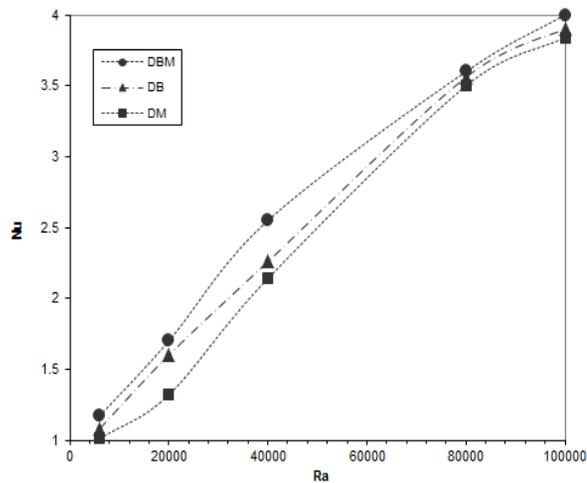


Fig. 3. Nusselt number obtained in case of DB, DM and DBM convection.

Over stable curves

Fig. 4 shows typical stability curves for DB and DM convection. These results correspond to the trend reported by Hashim and Wilson [4]. The latter investigation refers to a case which treats the effect of the surface tension as well.

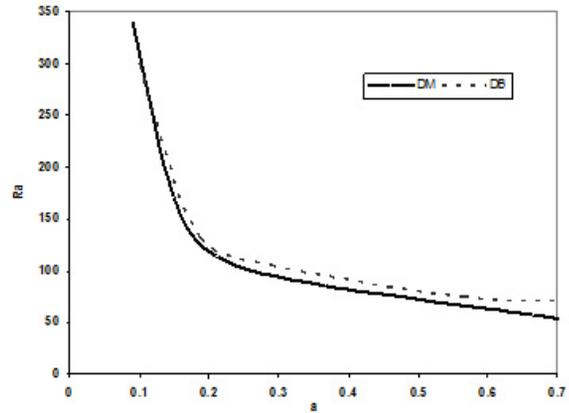


Fig. 4. Calculated stability curves referring to DB and DM convection.

CONCLUSIONS

Simulation of the onset of convection in a porous layer with concentration and temperature dependent viscosity is carried out. The Darcy model is employed in the mathematical formulation to evaluate the flow structure and the heat transfer in a two-dimensional fluid porous layer.

List of symbols

a	dimensionless wave number
c	concentration of diffusing fluid
De	effective diffusivity in porous medium
D	diffusivity in porous medium
d	tube diameter
g	gravity constant
\mathbf{g}	gravity vector
K	permeability
L	fluid layer thickness
Nu	Nusselt number
p	pressure
R	Rayleigh number
T	temperature
w	z-component of the velocity
W	dimensionless w
\mathbf{v}	velocity vector
z	vertical coordinate
Z	dimensionless vertical coordinate
α	coefficient of concentration expansion
α_T	coefficient of thermal expansion
β	viscosity-concentration dependence factor

β_T	viscosity-temperature dependence factor
μ	fluid viscosity
ρ	fluid density
τ	surface tension -temperature dependence factor
σ	surface tension

Subscripts

c	critical property
	property at reference concentration

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