INFLUENCE OF THE TRANSVERSAL DISPLACEMENTS OF METAL ON THE CAMBER FORMATION OF HOT-ROLLED STRIP

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Abstract

Transversal displacements of metal in the hearth of deformation reduce unevenness of elongations across strip width, wherein the camber curvature becomes smaller, and its radius is larger than that calculated on the assumption of plane deformation scheme.

The investigations are based on the variational principle of possible changes of strain state (Jourdain principle). It is assumed that the input velocity unevenness of the metal flow in the hearth of deformation across the strip width passes into an output one proportionally to reductions. Further, velocities and powers of deformation and displacements of metal in the hearth of deformation during hot rolling and corresponding powers are calculated.

Solving Euler-Poisson’s equation, we receive the expression for coefficients, which take into account the influence of the transversal displacements of metal on decreasing the output unevenness and the radius of camber curvature for every harmonic.

Keywords: hot-rolled strips, camber, wedge, metal transverse displacements, Jourdain variational principle, Cauchy-Bunyakovsky inequality, Euler-Poisson’s equation, strip’s cross-section.

Introduction

Consumer properties of hot rolled steel strips depend on many parameters, among which the most essential are such parameters as longitudinal and transverse thickness, camber and wedge of a cross sectional profile [1-13]. Hot-rolled strip’s camber formation has a great influence on straight movement of a strip when it fills the finishing group of a hot-rolling mill. Lateral movement of a hot rolled strip may be a reason of emergency situation in the mill (Fig. 1).

That is why the study of hot-rolled strip camber formation in the roughing group is very important.

Theoretical

At the entrance of the hearth of deformation the left edge of the rolled strip is thicker than the right one, and at the exit - the thickness of the strip is constant along the width. At uneven reduction of the strip across the width, transverse displacements of metal take place in the hearth of plastic deformation [1 - 2].

The metal transverse displacements in the hearth of plastic deformation decrease the unevenness of the lengthening coefficients and longitudinal stresses across the width of the rolled strips at the exit of the hearth of plastic deformation. This effect was suggested to be taken
into account with the help of the coefficient $\rho$ [1-2]:

$$\frac{\Delta \lambda(y)}{\lambda} = \rho \left( \frac{\partial h_0(y)}{h_{0m}} - \frac{\partial h_1(y)}{h_{1m}} \right)$$

(1)

$\Delta \lambda(y)$ and $\lambda$ are the value of the current non-uniformity of the elongations and the value of average elongation across the strip width, $\partial h_0(y)$ and $h_{0m}$ - the value of the current transverse thickness variation and the average thickness of rolled stock, $\partial h_1(y)$ and $h_{1m}$ - the value of the current transverse thickness variation and the average thickness of the strip, $0 < \rho < 1$ - coefficient taking into account the influence of transverse displacements of the metal in the hearth of plastic deformation.

In the case under consideration, the output transverse thickness variation $\partial h_1(y) = 0$, while the input transverse thickness variation $\partial h_0(y)$ changes linearly from $(+\partial h_m)$ on the left edge $(-B)$ to $(-\partial h_m)$ on the right $(+B)$, $B$ - semiwidth of the strip (Fig. 2).

On the entrance to the hearth of plastic deformation, unevenness of high-rise deformation, longitudinal tensions and velocities of metal flow across strip width is noted. Since the value of deformation and unevenness of velocities are significantly small in comparison with their average estimates, generally this unevenness can be expressed as unevenness of metal velocities on the entrance to the hearth of plastic deformation.

Let us describe input unevenness of velocities as $f'(y) < 1$, and output - $\varphi'(y) < 1$. Let us assume that the spreading when rolling is absent, i.e.

$$f(0) = f(B) = 0 \text{ and } \varphi(0) = \varphi(B) = 0, \text{ } B - \text{ semiwidth of a strip; in case of plane deformation scheme } f'(y) = \varphi'(y).$$

For the deformation hearth, we will assume the model of the rigid-plastic media with elastic external zones, i.e. we consider that a metal not possessing elasticity in the deformation hearth at once acquires it on the exit from the deformation hearth.

Applying the Jourdain variation principle to such defined deformation we get:

$$\delta \left[ \int_{\Omega} H d\Omega - \int_{S} \bar{\sigma} n \tilde{v} d\Omega + \tau_s \sum_{i} \int_{S_i} |\Delta v_i| d\Omega \right] = 0 \quad (2)$$

$H$ is the shear deformation rates intensity; $\bar{\sigma} n$ and $\tilde{v}$ - full external stresses operating on the surface of the deformation hearth $S$, and velocities of displacements corresponding to them; $\Omega$ - volume of the deformation hearth; $\tau_s$ - shear yield point of strip material; $|\Delta v_i|$ - jump of velocities on $i$-th shear surface $S_i$; $\delta$ - variation symbol.

The expression enclosed in parentheses is a functional and represents a rolling power $N_{roll}$. The first integral
represents the power of internal resistance, the second – the power of external forces acting on the deformation hearth borders, the third – the power of cut forces.

The expression for the longitudinal velocity distribution across the strip width can be written in the assumption that the input velocity unevenness of the metal flow in the hearth of deformation across the strip width passes into an output one proportionally to reduction:

$$v_x(y) = v_x\left[1 + f'(y)\frac{h_x - h_1}{\Delta h} + \phi'(y)\frac{h_0 - h_x}{\Delta h}\right]$$

where \(\Delta h = h_{0m} - h_1\) - absolute reduction,

\(h_x = h_1 + \Delta h\left(\frac{x^2}{\ell}\right)\) - current strip thickness in the hearth of deformation approximated by quadratic parabola; \(\ell\) - length of the hearth of deformation; \(v_x(y)\) - distribution of longitudinal velocities of metal across the strip width in the cross section \(x\), divided by the circumferential speed of the roll \(v_r\); \(\bar{v}_x\) - average value of velocities of metal across the strip width in cross-section \(x\), divided by the circumferential speed of the roll \(v_r\); \(\bar{v}_0\) - average value of strip input velocity, divided by the circumferential speed of the roll \(v_r\).

The mass flow stability principle (current strip thickness \(h_x\) is denoted by \(h\)) establishes that \(h_1 = h_0 - h\) taking into account that the average value of strain rate of metal across the strip width in cross-section \(x\):

$$\bar{\xi}_x = \frac{\partial \bar{v}_x}{\partial x} = -\frac{h_1 \bar{v}_1}{h^2} h' = -\frac{\bar{v}_x h'}{h}$$

From this formula:

$$\bar{v}_x = -\bar{\xi}_x \frac{h}{h'}$$

On the basis of (3)-(5) we obtain the expression for the strain rate \(\bar{\xi}_x\):

$$\bar{\xi}_x = \frac{\partial \bar{v}_x}{\partial x} = \bar{\xi}_x\left[1 + \frac{\phi' h_0 - f'h_1}{\Delta h}\right]$$

where \(\bar{\xi}_x = \frac{\partial \bar{v}_x}{\partial x}\) - average value of strain rate of metal across the strip width in cross-section \(x\).

Proceeding from the kinematic admissibility of velocities’ field on a roll surface, we receive

$$\frac{v_x}{v_z} = \tan \alpha = \frac{1}{2} \frac{dh}{dx} = \frac{h'}{2}.$$  

Taking into account that the value of high-rise deformation is constant, we receive \(\bar{\xi}_z = \frac{v_z}{h'}\). On a strip surface \(\bar{\xi}_z = \frac{v_x}{h'}\).

Taking into account (2) and (4) we receive

$$\bar{\xi}_z = -\bar{\xi}_x \left(f' - \phi'\right) \frac{h}{\Delta h}$$

From a condition of medium incompressibility we will receive the expression for strain rate along axis \(y\):

$$\bar{\xi}_y = \bar{\xi}_x \left(f' - \phi'\right)$$

The intensity of the strain rates \(\bar{\xi}\) is calculated neglecting the influence of shear deformation, using the expressions (5), (6) and (7):

$$H = 2\sqrt{\bar{\xi}_x^2 - \bar{\xi}_y \bar{\xi}_z} = \bar{\xi}_x \left[2 + f'\left(\frac{h - 2h_1}{\Delta h}\right) + \phi'\left(\frac{2h_0 - h}{\Delta h}\right) + \left(\phi'\right)^2\left(\frac{2h_0 - h}{(\Delta h)^2}\right) + \left(f'\phi'(\frac{-2h_0 h + \phi' h_1 h - 2h^2}{(\Delta h)^2}\right)\right]$$

In order to calculate the power of internal resistance, it is necessary to integrate the expression (8) in accordance with the equation (2) and the specificity of the chosen model of the medium:

$$\frac{N_{in} \tau_s}{\bar{v}_z} = \int_{-B}^{B} \int_{0}^{\ell} H dy dx dz$$
The power of the forces of sliding friction between the rolls and the strip:

\[
\frac{N_{sl}}{\tau_s} = 4\mu \int_{-B}^{B} dy \int_{0}^{L} [\Delta v_{sl}] dx = 4\mu \int_{-B}^{B} \sqrt{\Delta v_{1}^2 + v_{m}^2} dx
\]

where \( \Delta v_{sl} \) - sliding velocity of metal on the surface of the roll, \( \Delta v_{1} = v_{1} - 1 \) - sliding velocity of metal on the roll surface in the rolling direction divided by the circumferential speed of the roll \( v_{roll} \), \( \mu \) - friction coefficient.

After simple transformations we get:

\[
\frac{N_{sl}}{\tau_s} = 4\mu \int_{-B}^{B} dy \int_{0}^{L} [\Delta v_{1}^2 + \Delta v_{m}^2] dx
\]

\[
\int_{-B}^{B} \left[ \Delta v_{1} - \Delta v_{m} \right] dx = \int_{-B}^{B} \sqrt{\Delta v_{1}^2 + \Delta v_{m}^2} dx
\]

where \( \Delta v_{m} \) - half an amplitude of unevenness of metal velocities at the exit of the deformation hearth; \( \Delta v_{1} \) - current unevenness of metal velocities at the exit of the deformation hearth.

During one unit of time, the exit cross-section moves from the position 1-1 to the position 2-2 with the rotation angle and radius equal to, respectively, \( \phi \) and \( R_t \) (Fig. 4). It takes the power of rotation:

\[
N_t = M_t \cdot \omega_t
\]

where \( M_t \) is the strip bending moment; \( \omega_t \) - angular velocity of rotation of the cross section of the strip at the exit of the deformation hearth.

The cross-section rotation radius is determined as:

\[
R_t = B \frac{\bar{v}_1 - \Delta v_m}{\Delta v_m} = B \frac{\bar{v}_1}{\Delta v_m}
\]

The angular velocity of rotation of the cross section

\[
\omega_t = \frac{\Delta v_m}{B}
\]
\[ M_{\text{max}} = \sigma_{\text{max}} \cdot W \]  

where \( \sigma_{\text{max}} \) is the maximum tensile stress in the strip at the exit of the hearth of deformation; \( W \) - the resistant moment of the cross-section of the strip.

\[ W = \frac{J_z}{y_{\text{max}}} \]  

where \( J_z \) is the moment of inertia of the cross-section about the axis of rotation; \( y_{\text{max}} \) - the distance from the axis of rotation to the most stretched fibers of the strip.

\[ J_z = 4R_i B^2 h_i + 2R_i^2 B h_i + \frac{8}{3} h_i B^3 \]  

As \( \sigma_{\text{max}} \) we take its upper bound (yield point):

\[ \sigma_{\text{max}} = \sqrt{3} \cdot \tau_s \]  

Substituting (16) and (17) in (15) in view of (12) and neglecting small values, we get

\[ W = \frac{4B V_i^2 - B^2 h_i + 2B h_i \frac{B^2 V_i^2}{(\Delta v_m)^2} + \frac{8}{3} h_i B^3}{B \frac{V_i}{\Delta v_m}} = \frac{8}{3} h_i B^2 \frac{\Delta v_m}{V_i} + 2B^2 h_i \frac{V_m}{\Delta v_m} + 4B^2 h_i \]  

From (19), (14), (13) and (11) we get:

\[ N_i = \sqrt{3} \cdot \tau_s \left( \frac{8}{3} B h_i \frac{\Delta v_m}{V_i} + 2B^2 h_i \frac{V_i}{\Delta v_m} + 4B^2 h_i \right) \frac{\Delta v_m}{B} = \]

\[ = \sqrt{3} \cdot \tau_s \left[ \frac{8}{3} B h_i \cdot \frac{\Delta v_m}{V_i} + 2B h_i \cdot \left( \frac{\Delta v_m}{V_i} \right)^2 + 4B h_i \cdot \left( \frac{\Delta v_m}{V_i} \right) + 2B h_i \right] \]

We consider the design \( \frac{\Delta v_m}{V_i} \):

\[ \frac{\Delta v_m}{V_i} = \frac{V_i(B) - V_i(-B)}{V_i} = \phi'(B) - \phi'(-B) = \int_{-B}^{B} \phi''(y)dy \]

We rewrite the expression (20):

\[ N_i = \sqrt{3} \cdot \tau_s \left[ \frac{8}{3} B h_i \cdot \left( \int_{-B}^{B} \phi''(y)dy \right)^2 + 4B h_i \cdot \left( \int_{-B}^{B} \phi''(y)dy \right) + 2B h_i \right] \]

According to Cauchy-Bunyakovsky

\[ \left[ \int_{-B}^{B} \phi''(y)dy \right]^2 \leq 2B \cdot \int_{-B}^{B} \left[ \phi''(y) \right]^2 dy \]

in equality, we will write down:

\[ \frac{N_i}{\tau_s} = \sqrt{3} \left[ \frac{16}{3} B^2 h_i \cdot \frac{V_i}{B} \cdot \left( \int_{-B}^{B} \phi''(y)dy \right)^2 + 2B h_i \right] \]
Because Jourdain variational principle can be applied only to the mechanical systems in an equilibrium, then the considered hearth of deformation should be balanced. For this purpose at the exit of the hearth of deformation it is necessary to put the moment which provides the rectilinear movement of the strip and the power of which is numerically equal to the power of strip rotation. Thus the strip will carry away the power spent for the accumulation of potential energy:

\[ F = 4\mu t \left[ \frac{\Delta h}{h} \left( t^2 - t_0^2 \right)^{\frac{1}{2}} + \frac{2\nu h}{\ell \Delta h} t \cdot (\varphi - f) \right] dt + \]

\[ +\sqrt{\frac{16}{3}} B^2 h_1 \cdot \varphi \left( \varphi'(y) \right)^2 + 4\sqrt{3} Bh_1 \cdot \varphi'(y) + \]

\[ +\nu h_1 \frac{\left( \varphi'(y) \right)^2}{2} E' \]

Let us calculate the components of the equation (26):

\[ F_{\varphi} = 4\mu t \left[ \frac{\Delta h}{h} \left( t^2 - t_0^2 \right)^{\frac{1}{2}} + \frac{2\nu h}{\ell \Delta h} t \cdot (\varphi - f) \right] dt \]

\[ \frac{d}{dy} F_{\varphi} = \frac{d}{dy} \left[ \nu h_1 E' \cdot \varphi'' \right] \]

\[ \frac{d^2}{dy^2} F_{\varphi} = \frac{32}{3} \sqrt{3} h_1 \nu B^2 \cdot \varphi'''' \]

After simple transformation and simplification, we get the Euler-Poisson equation in the following form:

\[ \varphi - \frac{1}{K^2} \cdot \varphi'' + \frac{32\sqrt{3}}{3} B^2 \frac{1}{E'} \cdot \varphi'''' = f \]

Let us present the unevenness of output velocities of metal without taking into account the transverse movements in the hearth of deformation \( f'(y) \) and taking into account those \( \varphi'(y) \) in the following form:

\[ f'(y) = \sum_{i=1}^{n} A_i \sin \left( \frac{i\pi}{B} y \right) \]

\[ \varphi'(y) = \sum_{i=1}^{n} B_i \sin \left( \frac{i\pi}{B} y \right) \]

Substituting (31) and (32) in (30), we get

\[ B_i = \rho_1 \cdot A_i \]

where \( \rho_1 \) is the coefficient which takes into account the influence of transverse displacements of the metal in the hearth of plastic deformation on the reduction of unevenness of the \( i \)-th harmonic of the output speed of
the metal during the formation of a camber;

\[
\rho_i = \frac{1}{1 + \frac{1}{(KB)^2(\pi i)^2} + \frac{32\sqrt{3}}{3E}(i\pi)^2}
\]  

(34)

Thus, to estimate the actual camber of the rolled strip it is necessary to take into account the value of reduction of the unevenness of the thinning in the hearth of plastic deformation across the width in accordance with the expression (34).

For example, calculations show that when rolling strip \( h_{ou} = 50 \text{mm} \); \( h_{in} = 40 \text{mm} \); \( B = 1000 \text{ mm} \); \( \tau = 50 \text{MPa} \); \( \mu = 0.3 \) in work rolls with \( R = 450 \text{ mm} \), unevenness of metal velocities at the exit of the deformation hearth decreases almost four times in comparison with a plane deformation scheme because of transversal displacements of metal in the hearth of plastic deformation, and camber decreases as well.

**CONCLUSIONS**

With the help of the mathematical model developed we got a methodology that allows to calculate the decrease of the hot-rolled strip camber because of transverse displacements of metal in the hearth of plastic deform.

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